# Tightly Coupled Low-cost GNSS-RTK/INS/Odometer Integration Via Factor Graph Optimization Aided by GNSS Outlier Mitigation in Urban Canyons

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# Outline

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   - GNSS factor model
   - INS factor model
   - Odometer factor model
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6. Conclusion
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7. References

# Introduction

- Navigation in autonomous mobile robots (AMRs)
  - **Technic:** GNSS Real-time Kinematics (GNSS-RTK)
    - ➢ Definition: double-differenced GNSS localization (Takasu, 2009)
    - ➢ Function: centi-meter positioning in open-sky scenes
  - **Typical challenging**: GNSS signal degradation
    - ➢ Limited satellite numbers (Low availability)
    - ➢ Large measurement noise (Low accuracy)
  - **Ways to improve** navigation performance
    - Introduce complementary sensors
    - Use Kalman filter to cope with noise


Fig. 1. An autonomous mobile robot in campus

"There were many papers on **Kalman filter** based multi-sensors aided GNSS since 1980." (L.-T. Hsu, 2022, ION)

# Introduction

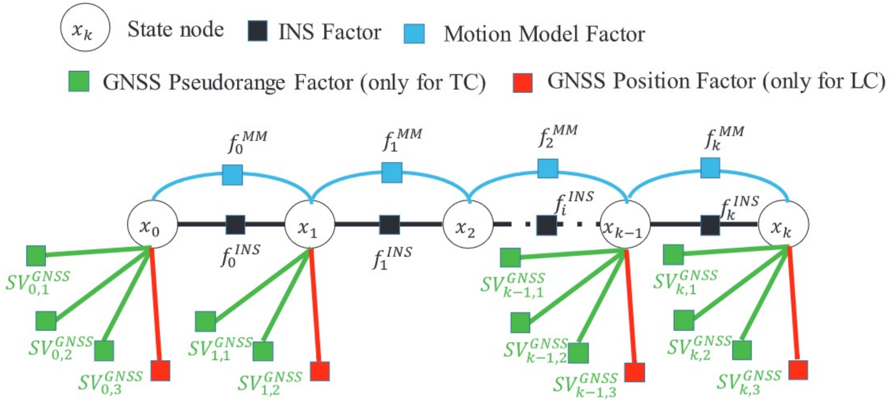
- The possibility: factor graph optimization (FGO) opens a new window
  - **Features of FGO**

**a) Pros**

- Keep batch measurements and states
- Apply iterative optimization
- …

**b) Cons**

- Heavy computing burden if the measurement amount is large
- …




"**Factor graph** based sensors-aided GNSS methods emerged since 2012." (L.-T. Hsu, 2022, ION)

# Introduction


**First FGO based GNSS positioning**

Pros: The first FGO-based GNSS localization.

Cons: Only employ pseudo-range measurements

**On manifold IMU pre-integration**

Pros: Utilize IMU in FGO.

Cons: Only apply in visual-inertial odometry.

**FGO based GNSS-PPP**

Pros: Employ carrier phase measurements.

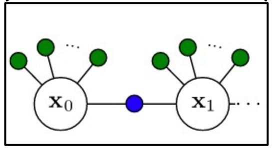
Cons: PPP needs time to converge.


**FGO based GNSS/INS integration**

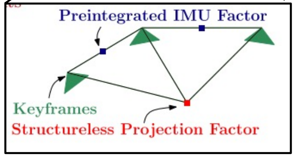
Pros: Develop loosely/tightly coupled GNSS/INS integration.

Cons: Limit to GNSS and IMU performance.

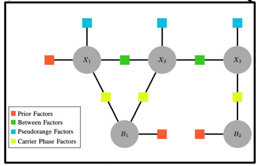
2012[1] 2016[2] 2018[3] 2021[4]



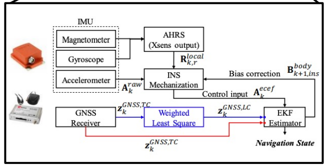

S. Niko,
P. Protzel, 2012




C. Forster,
et al., 2016




R. Watson,
et al., 2018




W. Wen, T. Pfeifer,
et al., 2021

[1] S. Niko, et al., *IEEE SSD*, 2012.
[2] C. Forster, et al., *T-RO*, 2016.
[3] R. Watson, et al., *PLANS*, 2018.
[4] W. Wen, et al. *NAVIGATION*, 2021

# Overview of the Proposed Method

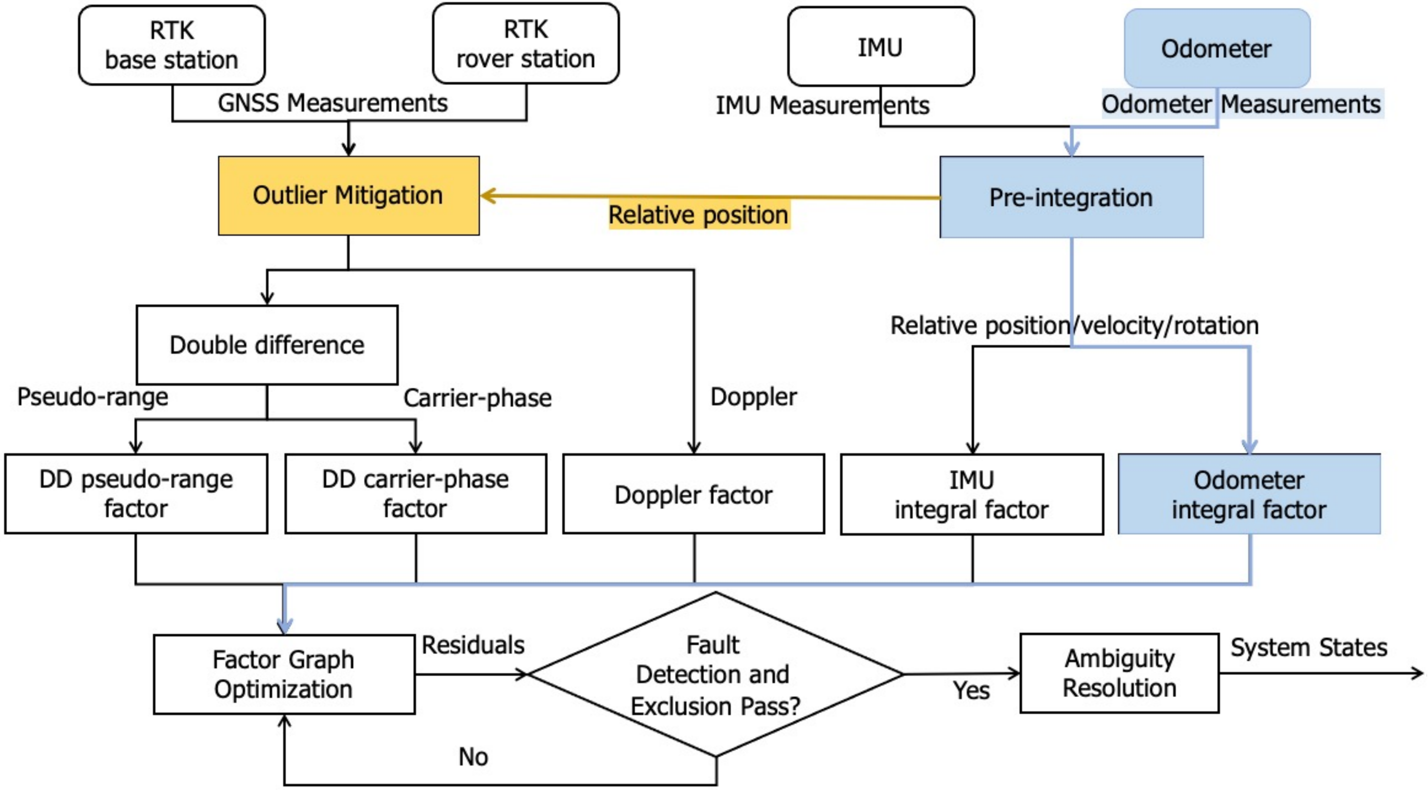



Fig. 2. The architecture of the proposed method

# Factor Graph Model

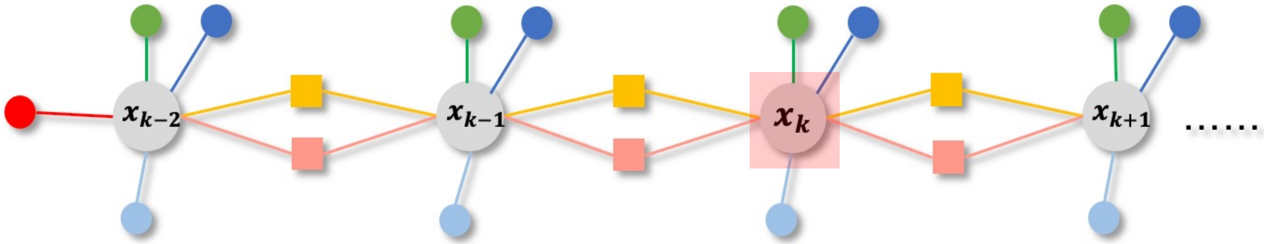



Fig. 3. The factor graph model

$$\boldsymbol{x_k} = \left[{}^{W}\boldsymbol{p}_{B,k}^{T}, \boldsymbol{q}_{B,k}^{W}{}^{T}, {}^{W}\boldsymbol{v}_{B,k}^{T}, \boldsymbol{b}_a, \boldsymbol{b}_g, d\dot{t}_r, s_v, s_\omega\right]^T$$

position | attitude | velocity | IMU bias | clock drift | Odometer scaler

| Symbols | Meanings | Symbols | Meanings |
|---|---|---|---|
| $e_{mar}$ | Prior marginalization factor | $e_{imu}$ | IMU pre-integration factor |
| $e_{\nabla\Delta P}$ | Double-differenced pseudo-range factor | $e_{odom}$ | Odometer pre-integration factor |
| $e_{\nabla\Delta L}$ | Double-differenced carrier-phase factor | $x_k$ | State node at epoch $k$ |
| $e_D$ | Doppler factor | | |

# Factor Graph Model

- GNSS factor model [4]
  - **Double differenced model**
    - ➢ $P_r^s = \rho + dT_r - dt^s + \varepsilon_P$
    - ➢ $\nabla\Delta P_{r,e}^{s,m} = (P_r^s - P_e^s) - (P_r^m - P_e^m) + \varepsilon_{\nabla\Delta P}$
  - **DD pseudo-range**
    - ➢ $e_{\nabla\Delta P} = \nabla\Delta P_{r,e}^{s,m} - \left(\nabla\Delta\rho_{r,e}^{s,m}\right)$
  - **DD carrier-phase**
    - ➢ $e_{\nabla\Delta L} = \nabla\Delta L_{r,e}^{s,m} - \left(\nabla\Delta\rho_{r,e}^{s,m} + \nabla\Delta N_{r,e}^{s,m}\right)$
  - **Doppler residual**
    - ➢ $e_D = D_r^s - \left(\frac{\Delta \boldsymbol{p}^T \Delta \boldsymbol{v}}{\rho} + d\dot{T}_r - d\dot{t}^s\right)$

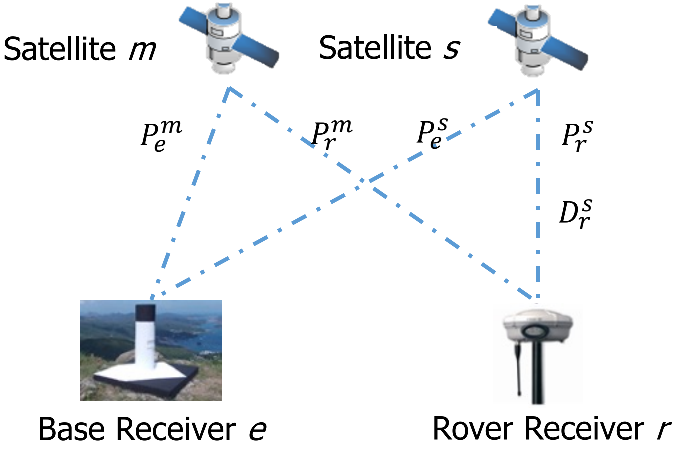



Fig. 4. The GNSS measurement model

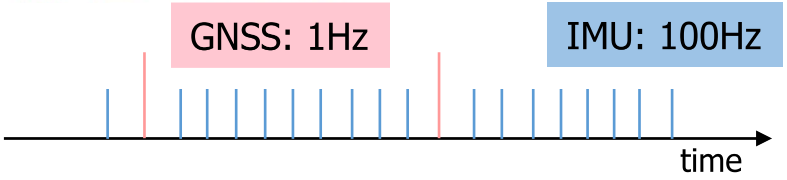
$\Delta \boldsymbol{p} = \boldsymbol{p}^s - {}^E\boldsymbol{p}_r$ : position vector

$\Delta \boldsymbol{v} = \boldsymbol{v}^s - {}^E\boldsymbol{v}_r$ : velocity vector

$\rho = \|\boldsymbol{p}^s - {}^E\boldsymbol{p}_r\|$ : geometry range

[4] W. Wen, et al. *NAVIGATION*, 2021

# Factor Graph Model




The high-rate IMU costs too much resource !

- IMU factor model [2]

- **Raw measurements from IMU**

$$a = \hat{a} + b_a - \varepsilon_a$$
$$\omega = \widehat{\omega} + b_g - \varepsilon_g$$

- $a$: true acceleration
- $\omega$: true angular velocity
- $b_a$: acceleration bias
- $b_g$: angular velocity bias

- **Pre-integration model:**

$$\alpha_{b_{k+1},imu}^{b_k} = \sum_{i=k}^{k+1}\left(\sum_{i=k}^{k+1}(\bar{R}_i^k(\bar{a}_i - b_{a,k} + \epsilon_{a,k})\delta t)\delta t\right)$$

$$\beta_{b_{k+1},imu}^{b_k} = \sum_{i=k}^{k+1}(\bar{R}_i^k(\bar{a}_i - b_{a,k} + \epsilon_{a,k})\delta t)$$

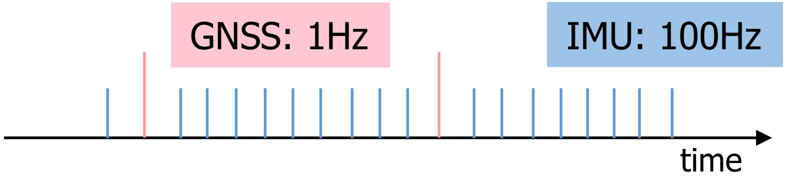
$$\gamma_{b_{k+1},imu}^{b_k} = \prod_{k}^{k+1}\left(\left(\begin{bmatrix}\bar{\omega}_i - b_{g,k} + \epsilon_{g,k} \\ 0\end{bmatrix}\right)\delta t\right)$$

- $\alpha_{b_{k+1},imu}^{b_k}$: integral position
- $\beta_{b_{k+1},imu}^{b_k}$:integral velocity
- $\gamma_{b_{k+1},imu}^{b_k}$: integral attitude

[2] C. Forster, et al., *T-RO*, 2016.

# Factor Graph Model

- IMU factor model [2]




- **Residual model**

$$
e^{k}_{imu,k+1} = \begin{bmatrix} \boldsymbol{\alpha}^{b_k}_{b_{k+1},imu} \\ \boldsymbol{\beta}^{b_k}_{b_{k+1},imu} \\ \boldsymbol{\gamma}^{b_k}_{b_{k+1},imu} \\ 0 \\ 0 \end{bmatrix} \boxminus \begin{bmatrix} R^{b_k}_{w}\left(p^{w}_{b_{k+1}} - p^{w}_{b_k} - v^{w}_{b_k}\delta t + \frac{1}{2} g^{w}\delta t^{2} - J^{\alpha}_{b_a}\cdot \delta b_{a,k} - J^{\alpha}_{b_g}\cdot \delta b_{g,k}\right) \\ R^{b_k}_{w}\left(v^{w}_{b_{k+1}} - v^{w}_{b_k} + g^{w}\delta t - J^{\beta}_{b_a}\cdot \delta b_{a,k} - J^{\beta}_{b_g}\cdot \delta b_{g,k}\right) \\ q^{b_k}_{w} \otimes q^{w}_{b_{k+1}} \otimes \begin{bmatrix} 0 \\ \frac{1}{2} J^{\gamma}_{b_g}\cdot \delta b_{g,k} \end{bmatrix}^{-1} \\ b_{a,k+1} - b_{a,k} \\ b_{g,k+1} - b_{g,k} \end{bmatrix}
$$

$J^{\alpha}_{b_a}, J^{\alpha}_{b_g}, J^{\beta}_{b_g}, J^{\beta}_{b_g}, J^{\gamma}_{b_g}, J^{\gamma}_{b_g}$: Jacobian from integral position/velocity/attitude to accelerator/gyroscope bias

[2] C. Forster, et al., *T-RO*, 2016.

# Introduction

- The opportunity: FGO-based RTK/INS/Odometer (RIO) integration
  - **Definition**
    - "2D linear/angular velocity measurements from an odometer are perfectly suitable for differential-drive robots." (X. Zuo, 2019, ISRR)
  - How to construct FGO-based RIO? **(First contribution)**
    - **Preprocess**: Develop modified pre-integration;
    - **Residual**: Derive Odometer residual model in the tightly coupled model;
    - **Jacobian**: infer the first-order derivative of the full state

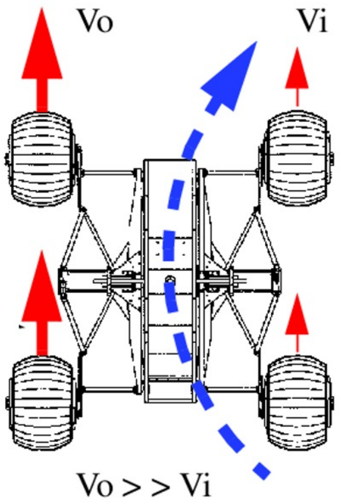
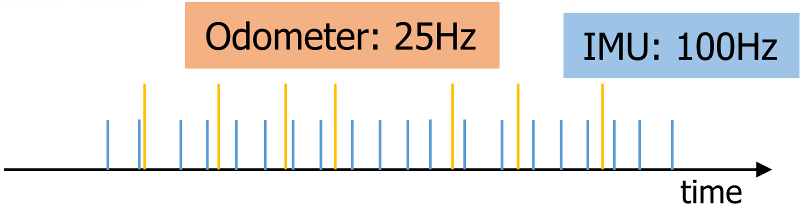



Fig. 5. Differential-drive model ($V_o/V_i$: velocity on one wheel)

# Factor Graph Model

- Odometer factor model




The Odometer output rate is also high!

- **Raw measurements from Odometer**

$$v^b = (1 + s_v) \cdot \hat{v}^b - \varepsilon_v$$
$$\omega^b = (1 + s_\omega) \cdot \hat{\omega}^b - \varepsilon_\omega$$

$v^b$: true velocity

$\omega^b$: true angular velocity

$s_v$: velocity scaler

$s_\omega$: angular velocity scaler

- **Modified Pre-integration model:**

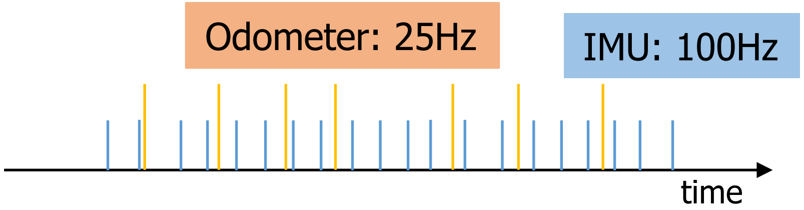
$$\alpha_{b_{k+1},odo}^{b_k} = R_w^{b_k} \int_k^{k+1} R_{b_t}^{w} \left( (1 + s_v)\hat{v}^b + \varepsilon_v \right) dt$$
$$\gamma_{b_{k+1},odo}^{b_k} = \int_k^{k+1} \frac{1}{2} \Omega' \left( (1 + s_\omega)\hat{\omega}^b + \varepsilon_\omega \right) q_t^{b_k} dt$$

$\alpha_{b_{k+1},odo}^{b_k}$: integral position

$\gamma_{b_{k+1},odo}^{b_k}$: integral attitude

# Factor Graph Model

- Odometer factor model




- **Residual model**

$$
\boldsymbol{e}^{k}_{odom,k+1} = \begin{bmatrix} \boldsymbol{\alpha}^{b_k}_{b_{k+1},odo} \\ \boldsymbol{\gamma}^{b_k}_{b_{k+1},odo} \\ 0 \\ 0 \end{bmatrix} \boxminus \begin{bmatrix} \boldsymbol{R}^{b_k}_{w}\left(\boldsymbol{p}^{w}_{b_{k+1}} - \boldsymbol{p}^{w}_{b_k} - \boldsymbol{J}^{p}_{s_v}\cdot\delta s_{v,k} - \boldsymbol{J}^{p}_{s_\omega}\cdot\delta s_{\omega,k}\right) \\ \boldsymbol{q}^{b_k}_{w}\otimes\boldsymbol{q}^{w}_{b_{k+1}}\otimes\begin{bmatrix} 0 \\ \frac{1}{2}\boldsymbol{J}^{\theta}_{s_\omega}\cdot\delta s_{\omega,k} \end{bmatrix}^{-1} \\ s_{v,k+1} - s_{v,k} \\ s_{\omega,k+1} - s_{\omega,k} \end{bmatrix}
$$

$\boldsymbol{J}^{p}_{s_v}$: Jacobian from integral position to velocity scaler

$\boldsymbol{J}^{p}_{s_\omega}$: Jacobian from integral position to angular rate scaler

$\boldsymbol{J}^{\theta}_{s_\omega}$: Jacobian from integral attitude to angular rate scaler

# GNSS Outlier Mitigation

- Introduction of sensor-aided GNSS outlier mitigation (OM)
  - **Definition**
    - "To get the best performance out of GNSS in urban areas, it is necessary to minimise the impact of NLOS reception and multipath interference on the position solution" (P. Groves, 2013, ION GNSS+)
  - Pre-integration aided GNSS OM **(second contribution)**

Predicted range from pre-integration
Real pseudo-range and carrier phase
⇨ Compute detection value ⇨ Outlier mitigation

# GNSS Outlier Mitigation

- Introduction of sensor-aided GNSS outlier mitigation (OM)
  - **Real pseudo-range and carrier-phase**

$$\delta L_{GNSS,k+1} = \nabla\Delta L_{k+1} - \nabla\Delta L_k$$

  - **Predicted range from pre-integration**

$$\delta L_{pre,k+1} = \nabla\Delta\hat{\rho}_{pre,k+1} - \nabla\Delta\hat{\rho}_{,k}$$

  - **Compute detection value :**

$$\kappa_{k+1} = |\delta L_{pre,k+1} - \delta L_{GNSS,k+1}|$$

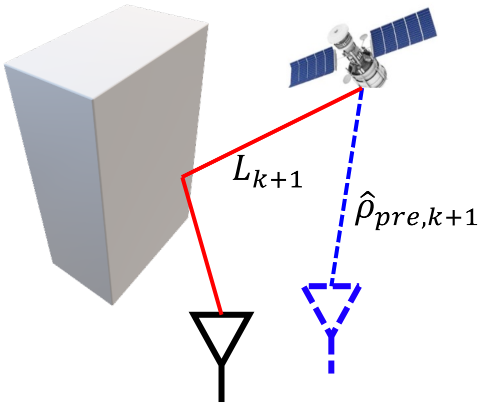



Fig. 6. the real GNSS measurement and the predicted range

# Numerical Experiments

- Experiment platform
  - **Evaluating Sensor (Low-cost)**
    a) GNSS: U-blox F9P receiver + Beitian-300S antenna
    b) IMU: MPU9250 (gyro bias stability 180 $°/h$)
    c) Odometer: Nidec encoder (100 pulse per round)
  - **Ground-Truth Sensor (High-cost)**
    d) LiDAR: Velodyne HDL-32E (vertical resolution 32 line)
    e) IMU: Xsens Mti-30 (gyro bias stability 18 $°/h$)
  - **Robot Platform**
    - UrsRobot Nexmow M1 (operation speed 0.2 - 0.7 m/s)
    - Software: ubuntu 20.04, ROS noetic, Ceres-solver 2.1.0

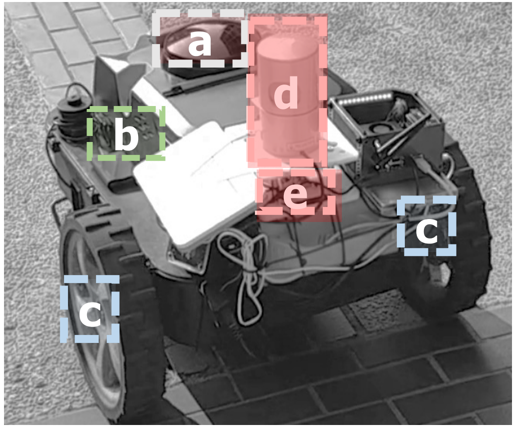



Fig.7. Experiment platform

How to make it?

Credit: https://ursrobot.ai/product-category/products/nexmow-robot-mower/

# Numerical Experiments

- Tightly coupled LiDAR-inertial odometry (LIO-SAM[5]) with loop closure
  - **Input**: IMU data and lidar point clouds
  - **Output**: keyframe positions in local frame
  - **Method features**

    a) Accuracy analysis

    ➢ High precision point cloud (2 cm)

    ➢ Tightly coupled via factor graph

    b) Alignment between ECEF/local frame

    ➢ Use 30% trajectory to compute the transform between ECEF/local frame[6-7]

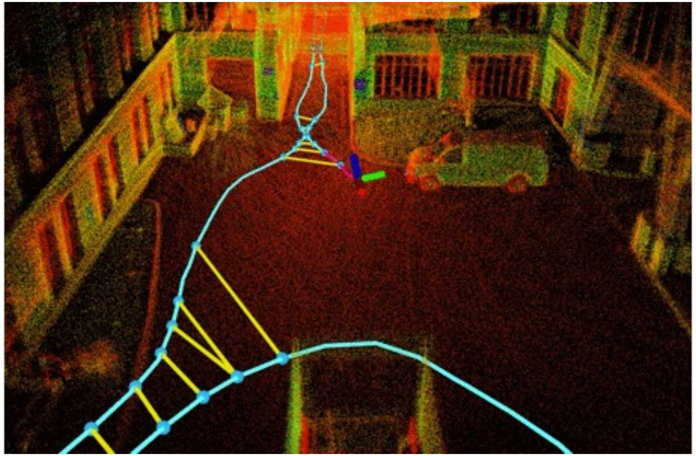


Fig. 8. LIO-SAM with loop closure[1]

[5] Shan, T. et al., IROS, 2020. [6] Zhang, Z. et al., IROS, 2019. [7] Delmerico J. et al., ICRA, 2018.

# Numerical Experiments

- Compared methods

| Schemes | GNSS | IMU | Odometer | Outlier Mitigation (OM) |
|---|---|---|---|---|
| RTKLIB-RTK [8] | ✓ | x | x | G* |
| F9P-RTK [9] | ✓ | x | x | G |
| RI [4] | ✓ | ✓ | x | G |
| RI-OM | ✓ | ✓ | x | GI** |
| RIO | ✓ | ✓ | ✓ | G |
| RIO-OM | ✓ | ✓ | ✓ | GIO*** |

*"G" means only using GNSS;
**"GI" means using GNSS and INS-aided outlier mitigation;
***"GIO" means using GNSS and INS/Odometer-aided outlier mitigation.

[8] T. Takasu et al., International Symposium on GPS/GNSS, 2009 [9] U-blox, 2023 [4] Wen, W. et al.. Navigation 2021

# Numerical Experiments

- Open-sky positioning performance
  - **Experiment design**
    a) satellite number/PDOP
    ➢ The average satellite number is 16.78
    ➢ The average PDOP is 1.34

    b) operation process
    ➢ Sidestep three laps around the lawn
    ➢ Simulate the regular movement of lawn mowers


Fig. 9. Open-sky trajectory

# Numerical Experiments

- Open-sky positioning under simulated GNSS outage
  - Absolute translation error (ATE)

Improve 69~93%

| GNSS outage (s) | 10 | 20 | 30 | 60 |
|---|---|---|---|---|
| RI-OM (m) | 4.86 | 16.40 | 38.55 | 168.93 |
| RIO-OM (m) | 1.49 | 3.96 | 5.79 | 10.83 |
| Improvement by RIO-OM | 69.34% | 75.85% | 84.98% | 93.59% |

➢ Integration of Odometer can improve the positioning performance in GNSS outage.

# Numerical Experiments

- Urban canyon positioning performance
  - **Experiment design**
    a) satellite number/PDOP
    - ➢ The average satellite number is 9.57
    - ➢ The average PDOP is 2.85

    b) operation process
    - ➢ Sidestep two laps around the lawn
    - ➢ Simulate the regular movement of lawn mowers

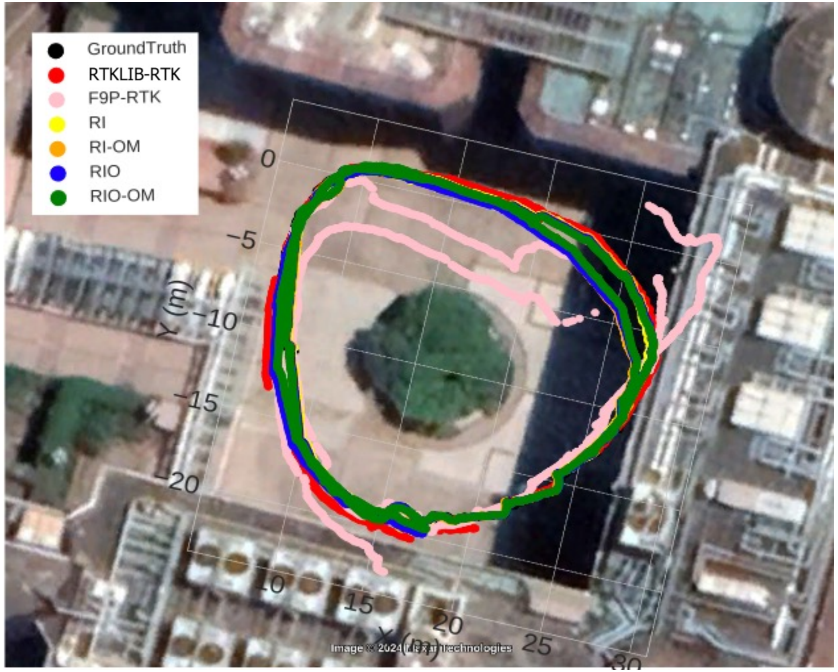



Fig. 10. Urban canyon trajectory

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香港理工大學

# Numerical Experiments

- Urban canyon positioning performance
  - Absolute translation error (ATE)

| 3D ATE | RTKLIB-RTK | F9P-RTK | RI | RI-OM | RIO | RIO-OM |
|---|---|---|---|---|---|---|
| MAX (m) | 3.86 | 11.95 | 1.72 | 0.95 | 1.03 | **0.85** |
| RMSE (m) | 1.44 | 4.71 | 0.76 | 0.38 | 0.52 | **0.30** |
| MAE (m) | 1.06 | 3.74 | 0.53 | 0.30 | 0.40 | **0.24** |

➢ RIO-OM reaches a RMSE of 0.30 m, with 79.2% and 93.6% improvement compared to RTKLIB-RTK and F9P-RTK.

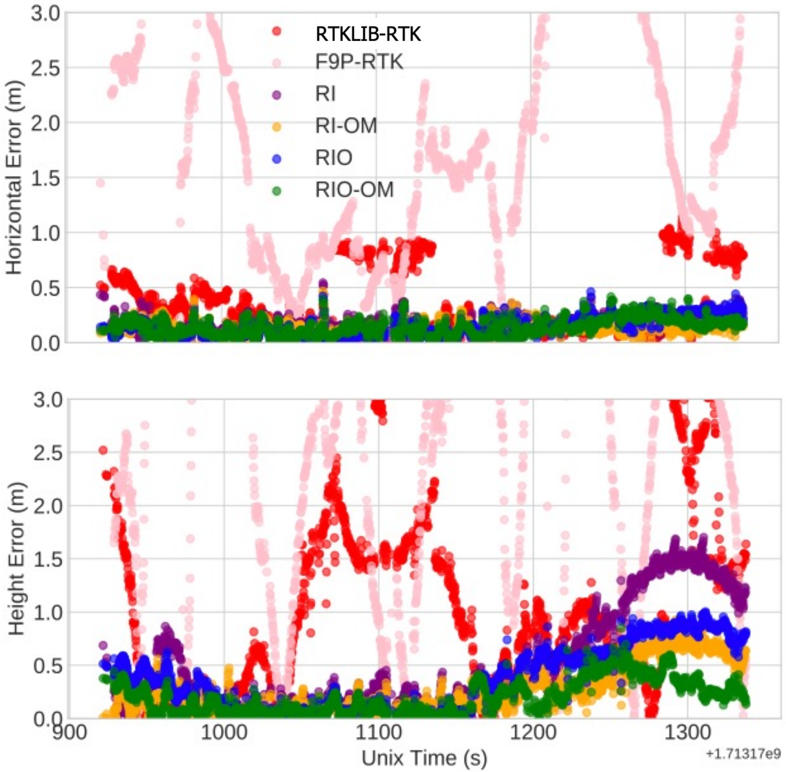


Fig. 11. Horizontal and height ATE

# Numerical Experiments

- Urban canyon performance (RIO-OM with sliding window size = 3)

| Platform | Personal Computer | Nvidia Orin-Nano | Nvidia Jetson-Nano | Raspberry-Pi 4B |
|---|---|---|---|---|
| **CPU** | Intel Core i9-9900K @ 3.6 GHz * 16 | ARM Cortex-A78AE @ 1.5 GHz * 6 | ARM Cortex-A57 @ 1.43 GHz * 4 | ARM Cortex-A72 @1.5GHz *4 |
| **RAM** | 32 GB LPDDR5 | 8 GB LPDDR5 | 4 GB LPDDR4 | 2 GB LPDDR4 |
| **CPU Usage** | 15.83 (using 1 core) | 6.71 (using 6 core) | 12.49 (using 4 core) | 23.50 (using 4 core) |
| **Memory Usage** | 0.48 | 2.98 | 3.78 | 8.17 |

➢ RIO-OM can run at a low-cost device (Raspberry Pi, $59), and only use <25% CPU.

# Conclusion

- Summary
  - Develop a novel **FGO-based** GNSS-RTK/INS/Odometer tightly coupled integration framework
  - Apply INS/Odometer-aided **GNSS outlier mitigation** to the proposed FGO framework
  - **Evaluate** the proposed method using the dataset collected in both **open-sky** and **dense urban scenes**

- Contribution
  - Build the modified pre-integration model to fuse Odometer into factor graph optimization
  - Expand the sensors used to aid GNSS outlier mitigation

# Conclusion

- Limitation

1. **Offline** extrinsic parameters **calibration** in advance is not suitable for time-varying robotic systems
2. Limited by GNSS signal, **initial** global position and bearing are obtained in open-sky scenes **outdoors**
3. Focus on improving the positioning performance by the **motion** model

- Future work

1. **Online** temporal/spatial parameters **calibration** identification to capture accurate sensor transformation
2. Develop a flexible **initialization** method for robots to start up **indoors**
3. Integrate **environmental features** to further improve the localization and mapping performance

# References

[1] Sünderhauf, N., & Protzel, P. (2012, March). Towards robust graphical models for GNSS-based localization in urban environments. In *International Multi-Conference on Systems, Signals & Devices* (pp. 1-6). IEEE.

[2] Forster, C., Carlone, L., Dellaert, F., & Scaramuzza, D. (2016). On-manifold preintegration for real-time visual--inertial odometry. *IEEE Transactions on Robotics*, 33(1), 1-21.

[3] Watson, R. M., & Gross, J. N. (2018, April). Evaluation of kinematic precise point positioning convergence with an incremental graph optimizer. In *2018 IEEE/ION Position, Location and Navigation Symposium (PLANS)* (pp. 589-596). IEEE.

[4] Wen, W., Pfeifer, T., Bai, X., & Hsu, L. T. (2021). Factor graph optimization for GNSS/INS integration: A comparison with the extended Kalman filter. *NAVIGATION: Journal of the Institute of Navigation*, 68(2), 315-331.

[5] Shan, T., Englot, B., Meyers, D., Wang, W., Ratti, C., & Rus, D. (2020, October). Lio-sam: Tightly-coupled lidar inertial odometry via smoothing and mapping. In *2020 IEEE/RSJ international conference on intelligent robots and systems (IROS)* (pp. 5135-5142). IEEE.

[6] Zhang, Z., & Scaramuzza, D. (2018, October). A tutorial on quantitative trajectory evaluation for visual (-inertial) odometry. In *2018 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)* (pp. 7244-7251). IEEE.

[7] Delmerico, J., & Scaramuzza, D. (2018, May). A benchmark comparison of monocular visual-inertial odometry algorithms for flying robots. In *2018 IEEE international conference on robotics and automation (ICRA)* (pp. 2502-2509). IEEE.

[8] Takasu, T., & Yasuda, A. (2009, November). Development of the low-cost RTK-GPS receiver with an open source program package RTKLIB. In *International symposium on GPS/GNSS* (Vol. 1, pp. 1-6). Seogwipo-si, Republic of Korea: International Convention Center Jeju Korea.

[9] u-blox. (2023).u-blox F9P Product Manual. 1(1): 16. https://content.u-blox.com/sites/default/files/ZED-F9P_IntegrationManual_UBX-18010802.pdf (accessed on 8 May 2024).

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14th-17th Oct. 2024, Hong Kong

# The Fourteenth International Conference on Indoor Positioning and Indoor Navigation


Location information of devices in indoor environments has become a key issue for many emerging applications. **IPIN** brings together experts in electronics, surveying, and informatics, facilitating discussions, collaborations, and new technologies to address indoor positioning and indoor navigation. Starting with its first edition in 2010, IPIN has shown to be the forum of excellence to join researchers, system developers, and service providers in the area of indoor positioning and navigation.

## Call for paper:

Regular Paper Submission: 3 June 2024
Notification of Regular Papers Acceptance: 8 July 2024
WiP Paper Submission: 15 July 2024
Notification of WiP Papers Acceptance: 5 August 2024

## Interested topics include but are not limited to:

- Indoor maps and building model
- Indoor positioning, navigation, and tracking methods
- GNSS and Indoor/outdoor seamless navigation
- Robotics and autonomous systems
- Mobile computing
- AI and deep learning
- Related applications

**Time: 14th-17th October 2024**
**Venue: InterContinental Grand Stanford Hong Kong, East Tsim Sha Tsui, Kowloon, Hong Kong.**

Organised by The Hong Kong Polytechnic University, Department of Aeronautical and Aviation Engineering, in full in-person mode.

Please find more details at ***https://ipin-conference.org/2024/*** or contact ***ipin2024.hk@polyu.edu.hk***

**Looking forward to meeting you at IPIN2024 in Hong Kong!**

# Thank you for your attention!

## Questions are welcome.

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# Appendix A

- Open-sky positioning performance
  - Absolute translation error (ATE)

| 3D Error | RTKLIB -RTK | F9P-RTK | RI | RI-OM | RIO | RIO-OM |
|---|---|---|---|---|---|---|
| MAX (m) | 0.49 | 0.63 | 0.39 | 0.40 | **0.35** | **0.35** |
| RMSE (m) | 0.22 | 0.19 | 0.19 | 0.18 | **0.18** | **0.18** |
| MAE (m) | 0.20 | 0.17 | 0.17 | 0.17 | **0.16** | **0.16** |

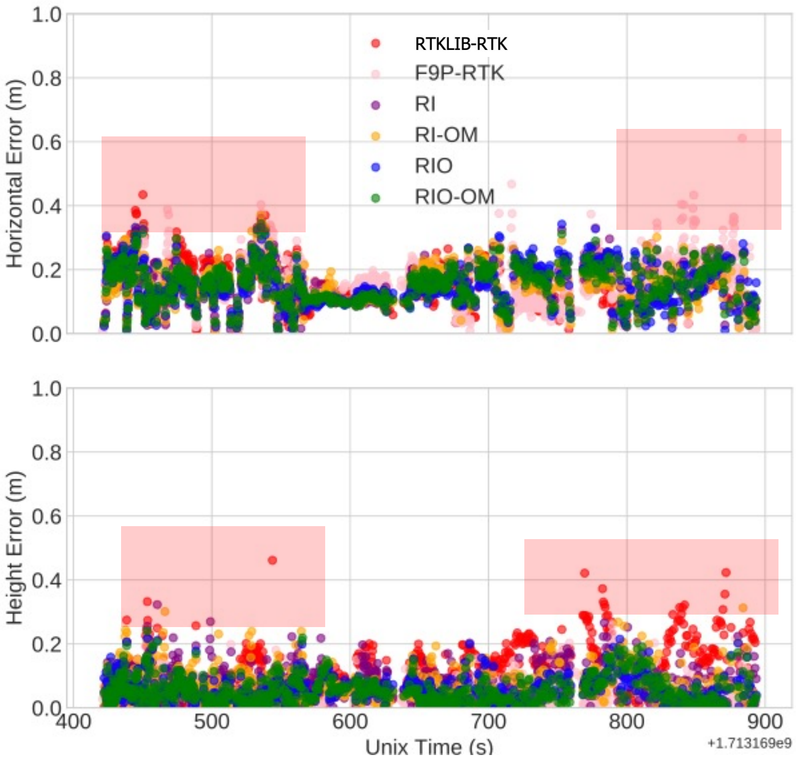


Fig. 12. Horizontal and Height ATE

➢ Integration of Odometer can help mitigate gross error in GNSS-RTK, according to MAX error;

# Appendix A

- Open-sky performance under simulated GNSS outage

Improve 69~93%

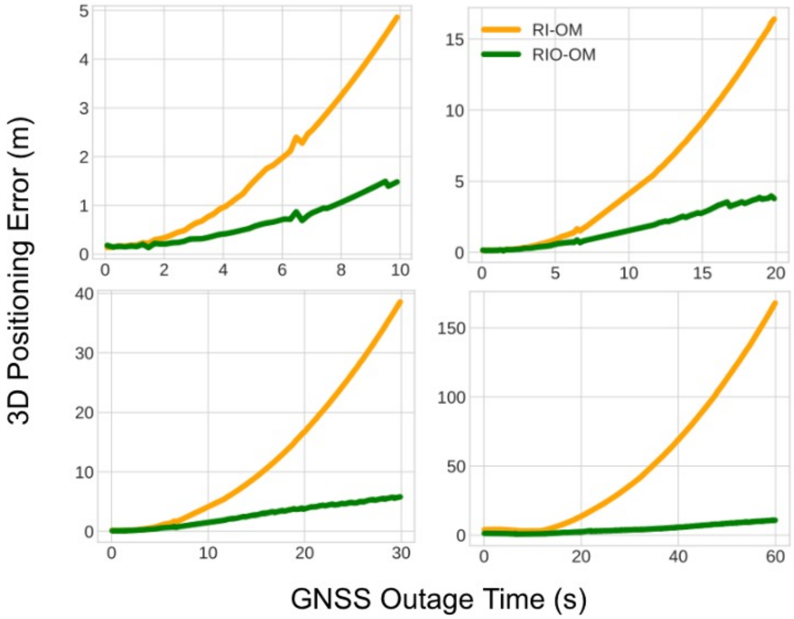


Fig. 12. 3D positioning ATE

| GNSS outage (s) | 10 | 20 | 30 | 60 |
|---|---|---|---|---|
| RI-OM (m) | 4.86 | 16.40 | 38.55 | 168.93 |
| RIO-OM (m) | 1.49 | 3.96 | 5.79 | 10.83 |
| Improvement by RIO-OM | 69.34% | 75.85% | 84.98% | 93.59% |

# Appendix B

- Urban canyon positioning performance

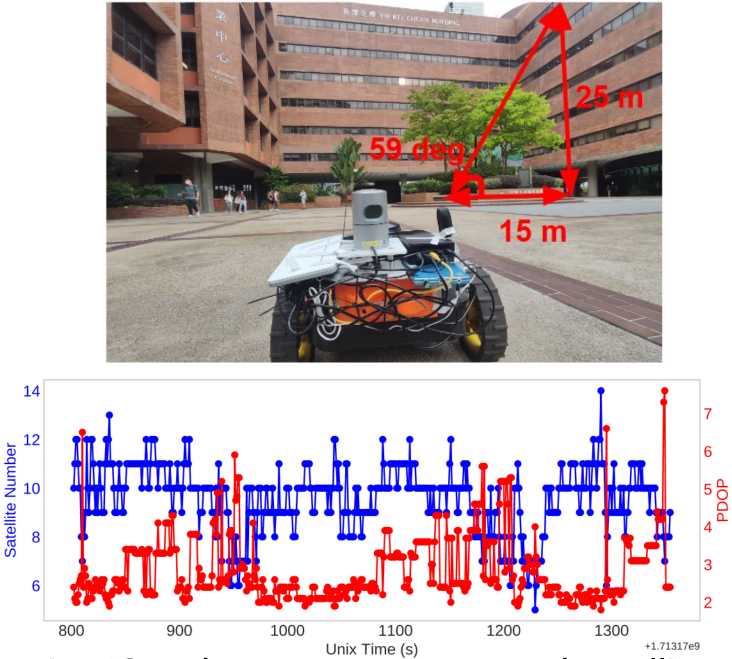


Fig. 13. Urban canyon scene and satellite number/PDOP

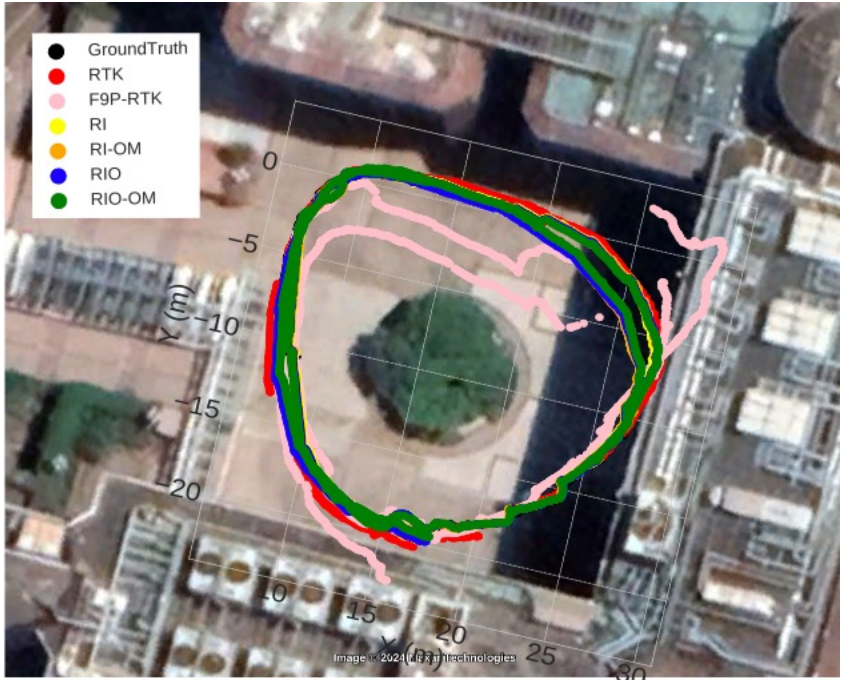


Fig. 14. Urban canyon trajectory

# Appendix C

- Introduction of sensor-aided GNSS outlier mitigation (OM)
  - **Error analysis**

a) Predicted range precision (local info)

b) Last position precision (global info)

➢ Assume measurements at last epoch without outliers

➢ Pre-integral position from IMU/Odometer is precise (3D ATE < 0.1 m in 1 second)

$$
\begin{aligned}
\delta\rho_{r,k+1}^{s} &= \rho_{r,k+1}^{s} - \rho_{r,k+1}^{s} \\
&= m_{k+1}\cdot p_{k+1} - m_k \cdot p_k \\
&= m_{k+1}\cdot (p_k - \delta p_r + \delta p^s) - m_k \cdot p_k \\
&= (m_{k+1} - m_k)\cdot p_k + m_{k+1}\cdot(-\delta p_r + \delta p^s)
\end{aligned}
$$

➢ $m_k$ is the line of sight vector at epoch $k$

➢ The sensor-aided GNSS outlier mitigation can tolerate large prior position error (up to 100 m error)