

# Baoshan Song (宋宝山)

(<https://baoshan-song.github.io/>)

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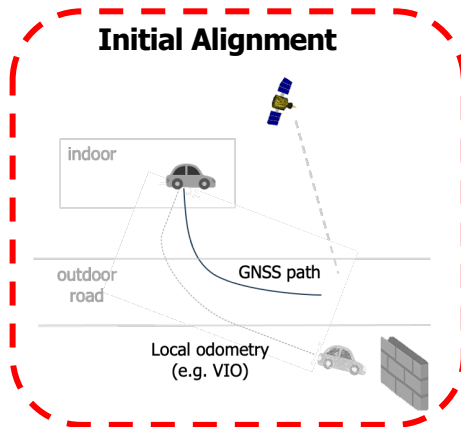


## Certifiable Navigation

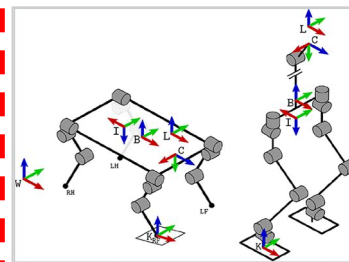
### Initial Positioning



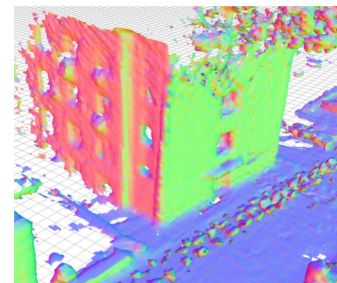
### Initial Alignment



### Sensor Calibration



### Localization and Mapping



# Certi fiable GNSS/Local Frame Alignment using Doppler measurements

**Baoshan Song**<sup>1</sup>, Dr. Matthew Giamou<sup>2</sup> and Dr. Li-Ta Hsu<sup>1</sup>

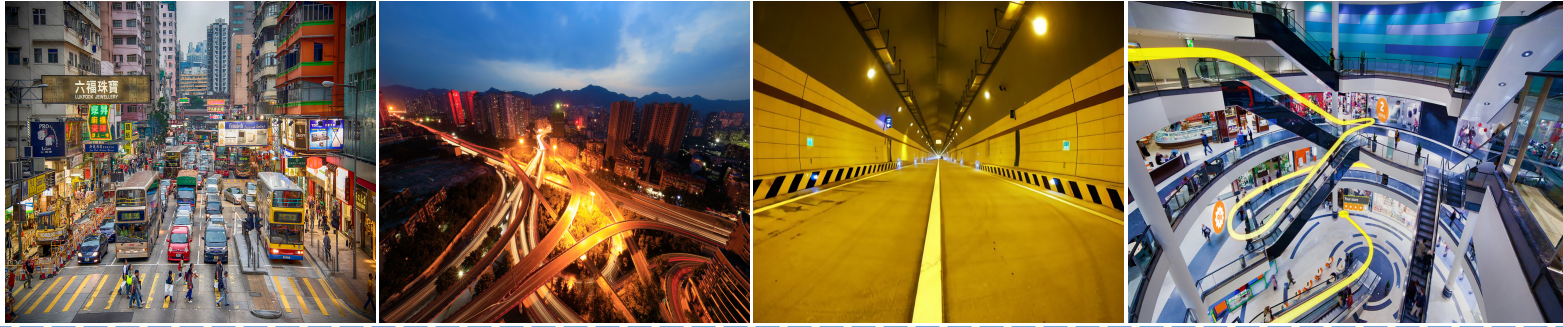
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[2] Autonomous Robotics & Convex Optimization Lab. McMaster University

Spe 29, 2025 @ PolyU Workshop, Hong Kong



# Toward Alpha plan: intelligent navigation

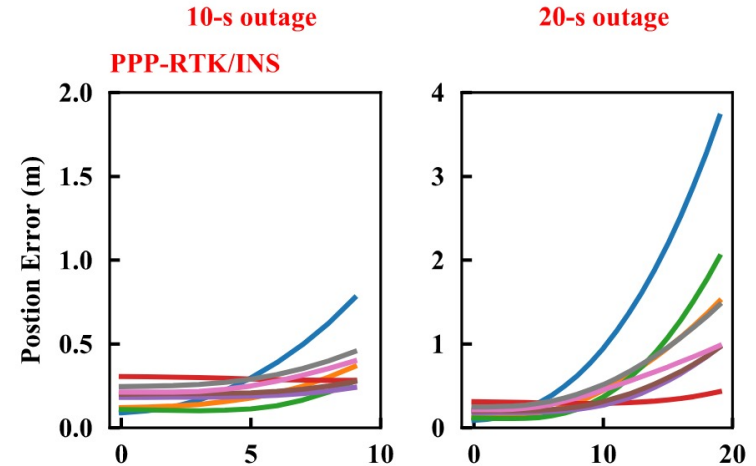


- ✓ **Various demand** in scenes and robots
- ✓ **Safe, precise and continuous** navigation is required
- ✓ **Seamless** navigation across all scenes is required

# Seamless navigation challenge: tunnel



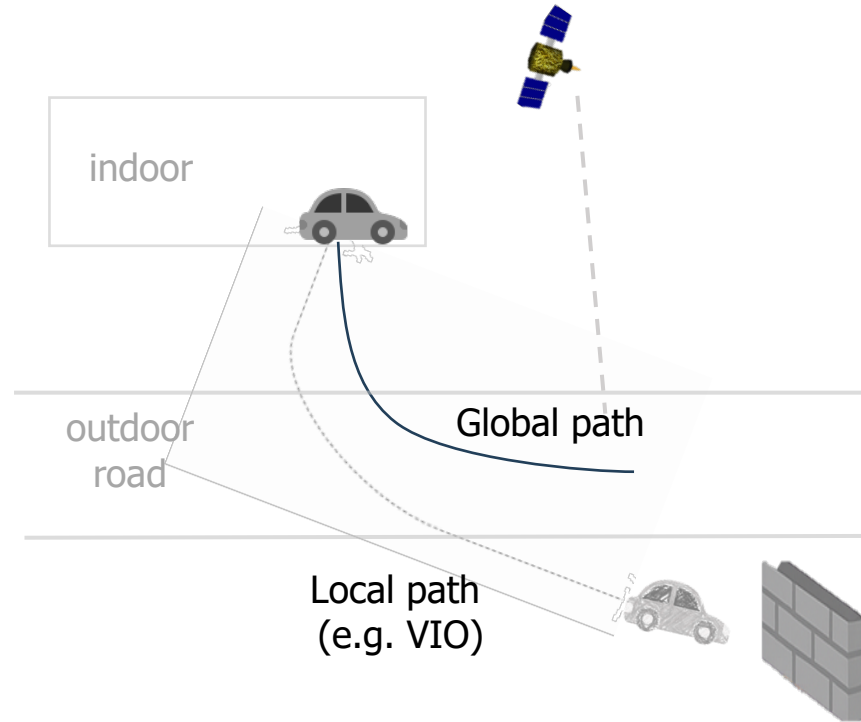
Driving through a tunnel



Position error with GNSS outage  
[Li et.al., 2022 *T-TITS*]

- ✓ A frequent initial alignment for GNSS/local frame is necessary!

# Seamless navigation challenge: urban parking



**Dangerous!**

Representative Work	Sensors	Input	Variable	Estimation	Dynamics	Initial guess
Gade(2016), Eling (2013)	GNSS	Multi-antenna	Yaw	Closed-form	Low	X
Wu (2011)	GNSS-IMU	Acceleration	Yaw	Closed-form	High	X
Zhang (2020)	GNSS-IMU	Velocity	Yaw	Closed-form	Low	X
Wei (2021)	GNSS-IMU	Doppler	Yaw	Closed-form	Low	X
Chen (2023)	GNSS-IMU	Position	Yaw	Closed-form	Low	X
Shin(2004), Groves(2013)	GNSS-IMU	motion	Yaw	Kalman Filter	Low	√
GVINS (2021)	GNSS-IMU- Visual	Doppler	Yaw	Local search	Low	√
P3-LINS (2022)	GNSS-IMU- LiDAR	Doppler	Yaw	Local search	Low	√
<b>Ours</b>	<b>GNSS-Local sensor</b>	<b>Doppler</b>	<b>3D rotation</b>	<b>Global search</b>	<b>Low</b>	X

Representative Work	Sensors	Input	Variable	Estimation	Dynamics	Initial guess
Gade(2016), Eling (2013)	GNSS	Multi-antenna	Yaw	Closed-form	Low	X
Wu (2011)	GNSS-IMU	Acceleration	Yaw	Closed-form	High	X

**Gap 1:** Rely on  
Abundant satellites

**Gap 2:** Require  
good initialization

Is it possible to obtain a guaranteed **optimal** estimate with **limited** satellite?

# Doppler based alignment

problem <sup>[1]</sup>:

$$\min_{\mathbf{R}_w^e \in SO(3), \bar{t} \in \mathbb{R}} \sum_{i=1}^K \|z_i\|_\epsilon^2$$

$$z_i = (\mathbf{R}_w^e \mathbf{v}_{r,i}^w - \mathbf{v}_{s,i}^e)^T \mathbf{n}_i + \bar{t} - \bar{D}_i$$

ECEF-world  
rotation

Local  
receiver  
velocity

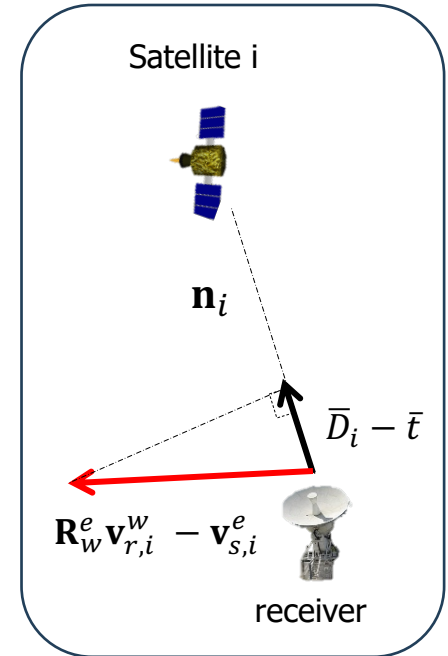
Satellite  
velocity

Line of  
sight  
vector

Receiver  
clock  
shift rate

Doppler  
measurement

- ✓ Current feasible region  $SO(3)$  is non-convex
- ✓ There are multiple local optima in a non-convex problem

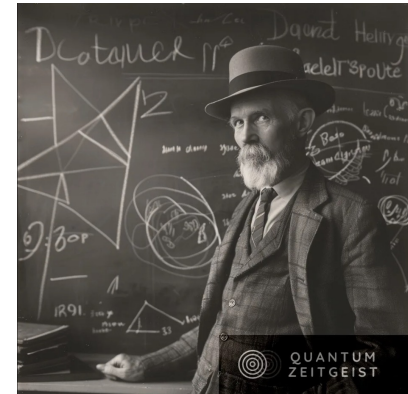


# How to guarantee optimality?

**Hilbert's 17<sup>th</sup> problem:** Can we rewrite a non-negative polynomial as sum of squares (SOS)?

**Answer:** [J. Lasserre, 2001]

For some problems, we can try to **guarantee optimality** of  $f(x)$  by testing the **positivity constraint**  $h(x)$  using SOS.



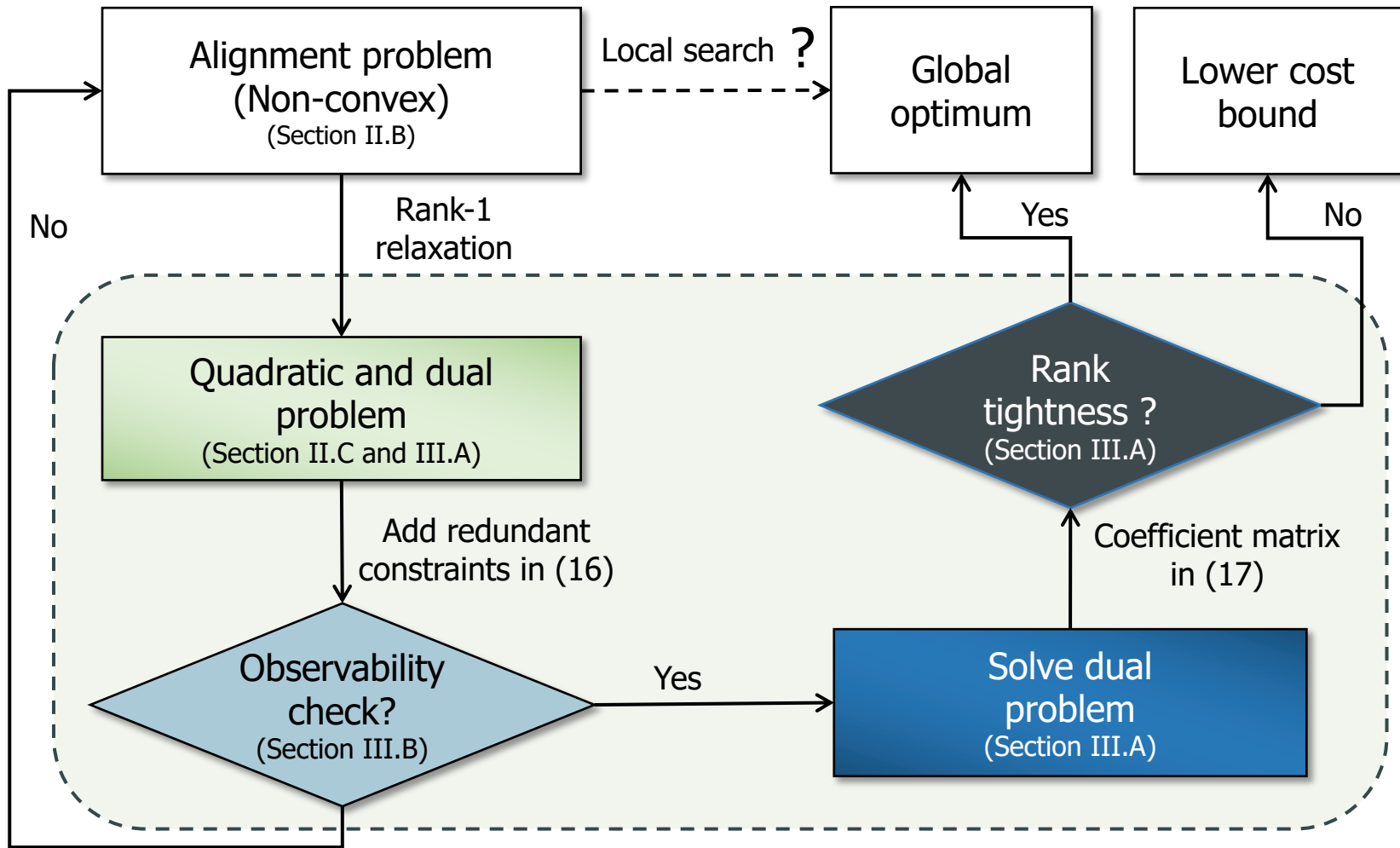
David Hilbert  
(Ref: QUANTUM)

$$\min f(x)$$



$$h(x) = f(x) - \gamma \geq 0 \quad \longrightarrow \quad h(x) = \sum_{i=1}^k g_i(x)^2 \quad \longrightarrow \quad h(x) \geq 0$$

with given lower bound  $\gamma$



# Step 1: problem reformulation

	Original (non-convex)	Quadratic (non-convex)	Dual (concave)
Variables	$\mathbf{R}_w^e \in SO(3), \bar{t} \in \mathbb{R}$	$\mathbf{r} = \text{vec}(\mathbf{R}_e^w) \quad \mathbf{x} = [\mathbf{r}^T \quad y]^T$	$\lambda, \gamma, M, N$
Cost function	$\min_{\mathbf{R}_w^e \in SO(3), \bar{t} \in \mathbb{R}} \sum_{i=1}^K \ z_i\ _c^2$	$\min_{\mathbf{x}} \mathbf{x}^T \bar{\mathbf{Q}} \mathbf{x}$	$\max_{\lambda, \gamma, M, N} \gamma$
Constraints	$z_i = (\mathbf{R}_w^e \mathbf{v}_{r,i}^w - \mathbf{v}_{s,i}^e)^T \mathbf{n}_i + \bar{t} - \bar{D}_i$  $SO(3) \equiv \{\mathbf{R} \in \mathbb{R}^{3 \times 3} :$  $\mathbf{R}^T \mathbf{R} = \mathbf{I}_3, \det(\mathbf{R}) = 1\}$	$s.t. \mathbf{R}^T \mathbf{R} = y^2 \mathbf{I}_3,$  $\mathbf{R} \mathbf{R}^T = y^2 \mathbf{I}_3,$  $\mathbf{R}^{(i)} \times \mathbf{R}^{(j)} = y \mathbf{R}^{(k)},$  $i, j, k = \text{cyclic}(1, 2, 3), y^2 = 1$	$s.t. \bar{\mathbf{Q}} + \mathbf{P}(\lambda, M, N, \gamma) \succeq \mathbf{0}$

Key insight here is to **solve the Dual problem** and **recover original solution**.

## Step 2: observability check

**Lemma 1** (Necessary conditions of observability with abundant constraints). *The problem for the dual problem with abundant constraints is observable with the Hessian matrix's degree of freedom (DOF)  $\geq 4$ . To obtain instantaneous alignment, we need at least 2 satellites and velocity along 2 axis.*

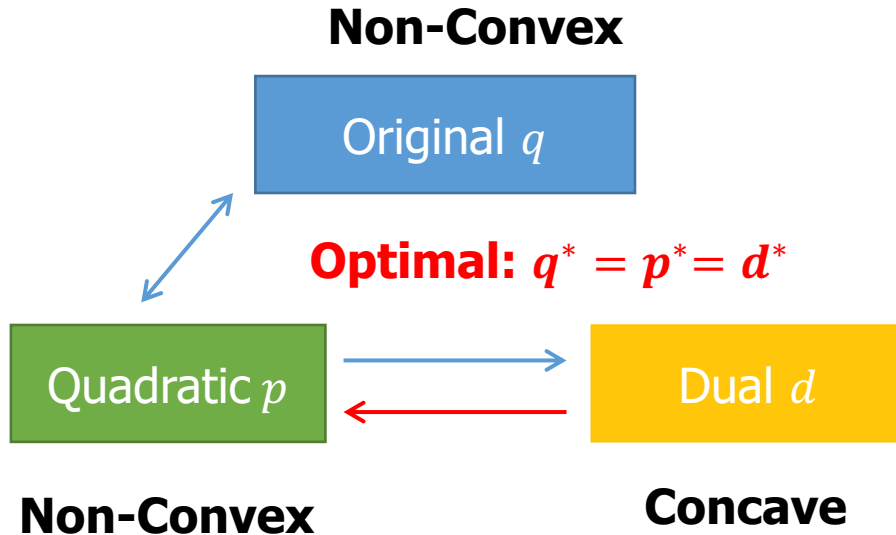
$$M_{1:K} = [\mathbf{v}_i \otimes \mathbf{n}_i]_{i=1}^K = \mathbf{V} \odot \mathbf{N}$$

Local receiver Velocity groups      Hadamard product      Line of sight vector groups

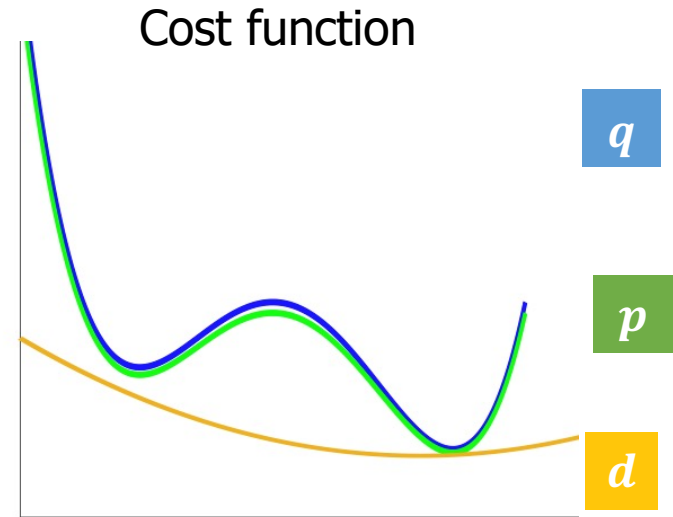
$4 \leq \text{rank}(\mathbf{M}) \leq \text{rank}(\mathbf{V}) \cdot \text{rank}(\mathbf{N})$

$\text{rank}(\mathbf{V}) \geq 2$  and  $\text{rank}(\mathbf{N}) \geq 2$

# Step 3: Optimality guarantees



Strong duality:  
Karush-Kuhn-Tucker (KKT) condition



Tight relaxation:  
Rank-1 condition

# Compared methods

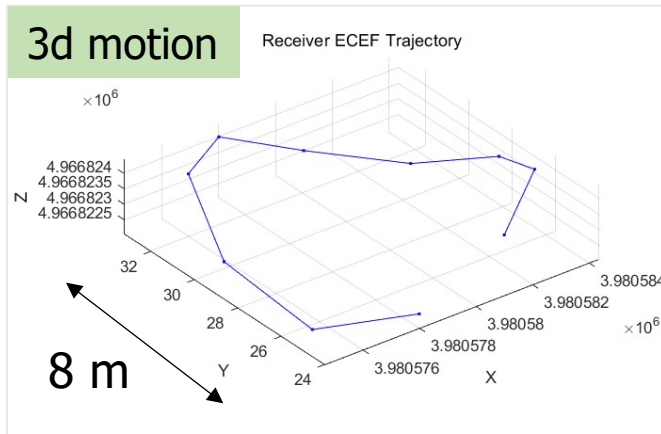
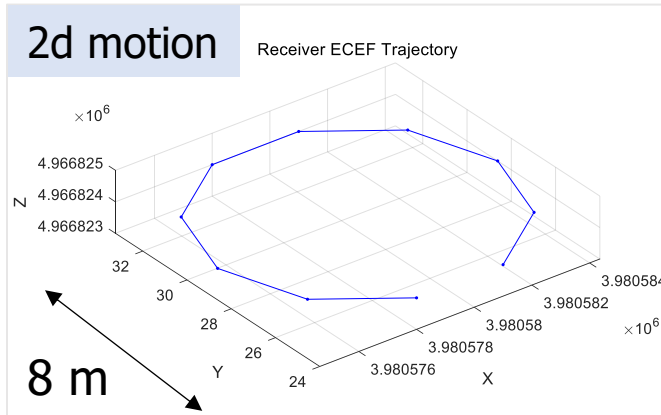
Configurations	VOBA <sup>[1]</sup>	GVINS <sup>[2]</sup>	Our method (SDP)
Solver	SPV+SVD	weighted least squares	CVX <sup>[3]</sup>
Iteration threshold	No	$10^{-6}$	Best precision
Solution type	Closed-form	Heuristic	Post-hoc Guaranteed
Initialization	No	Yes	No

[1] Zhang et.al., *IEEE SJ*, 2020

[2] Cao et.al, *T-RO*, 2021

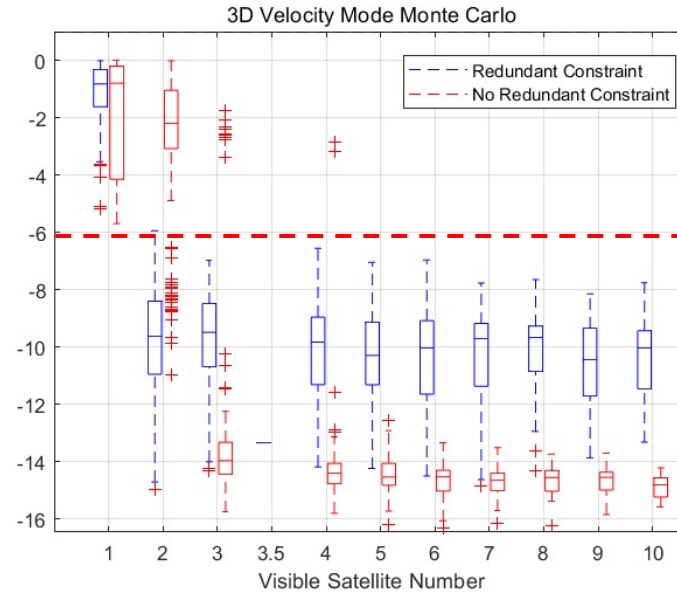
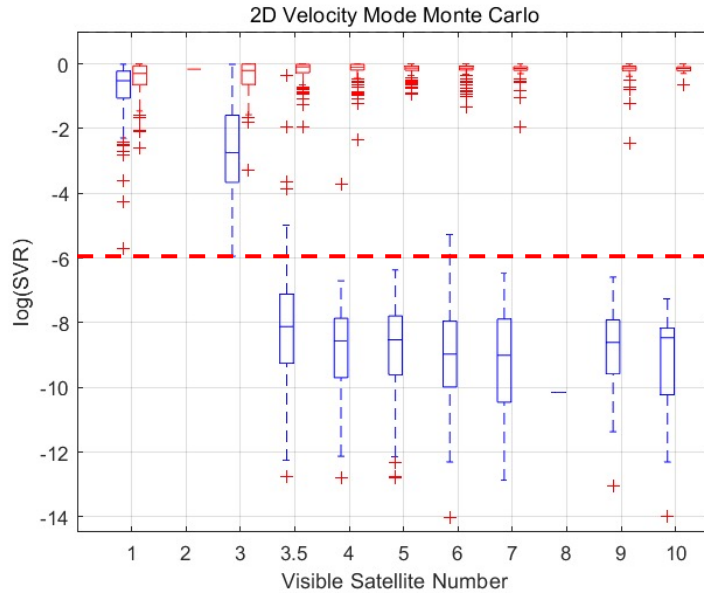
[3] Boyd, convex optimization, 2006

# Simulation tests



Settings	Value
Interval	1 second
Duration	10 seconds
Signal Frequency	1575.42 MHz
Receiver velocity	3 m/s
Orbit type	Walker
Clinical angle	55°
Elevation angle	≥10°

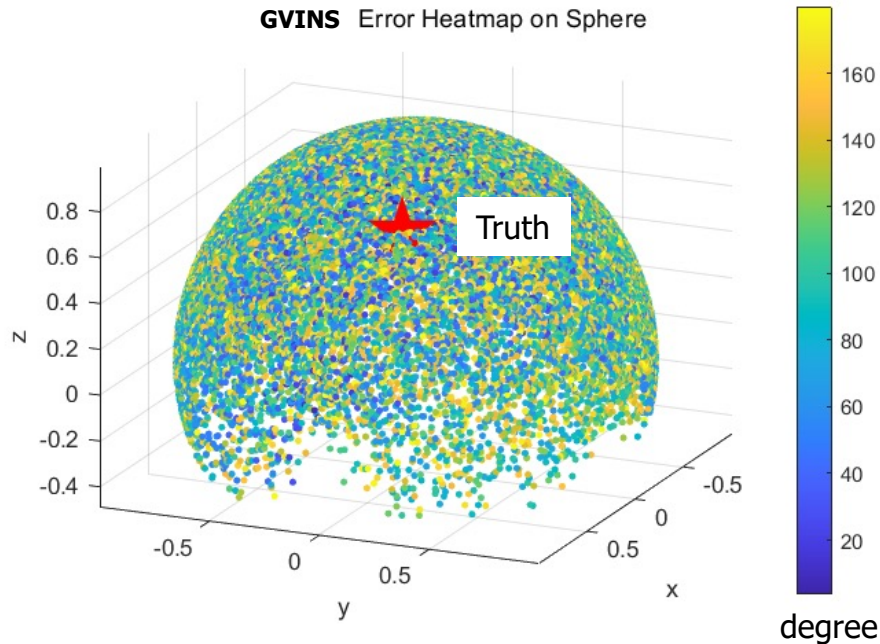
# Constraints v.s. Rank-tightness



optimal

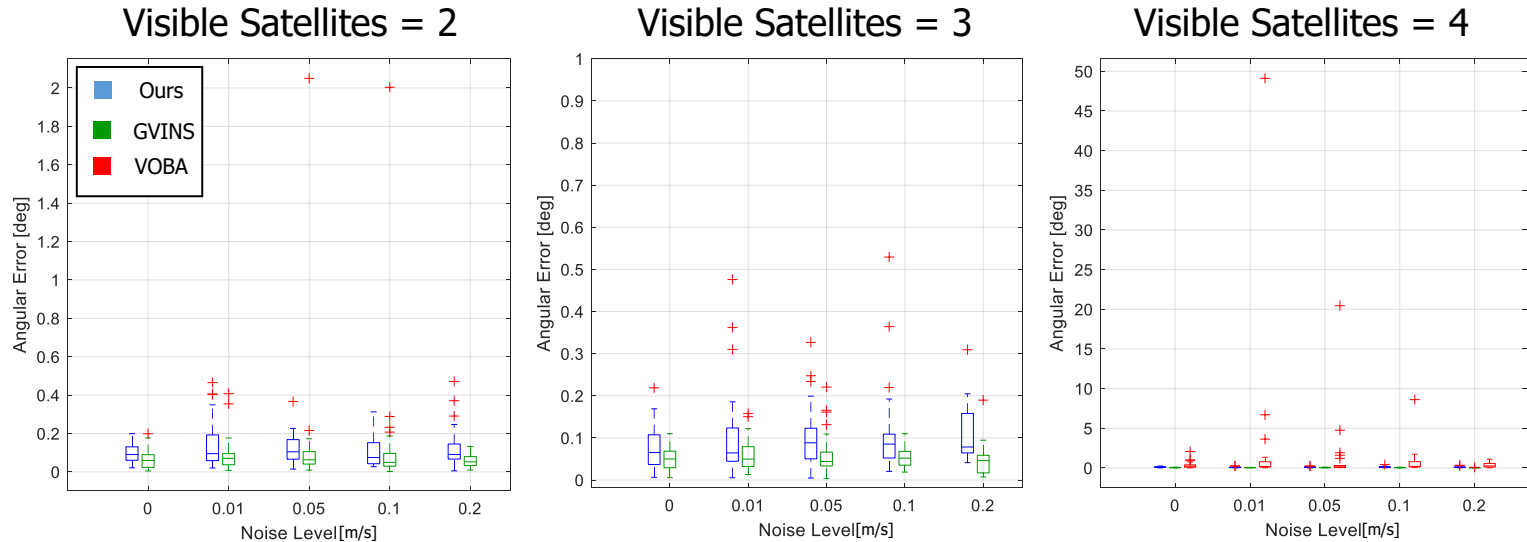
1. Redundant constraints increase the **robustness** of rank-tightness.
2. **Without abundant constraints, 2D motion** always leads to **failed** alignment.

# Initial guess v.s. Local searching method



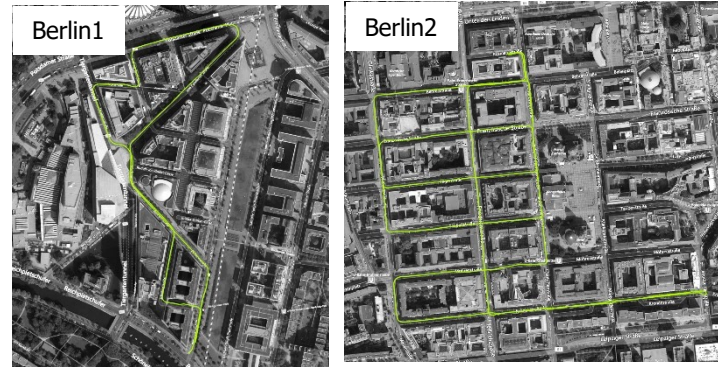
Under **poor observability**  
(visible satellite number = 2),  
Gauss-Newton is easy to be  
stuck in **local optima** with  
**random initialization**.

# Doppler noise v.s. Alignment errors



1. SDP provides certifiably optimal solution but the **precision** of SDP is worse than GN, which is caused by the solver.
2. **VOBA** outputs closed-form solution but is fragile and only work when the **satellite number**  $\geq 4$ .

# Real world tests



	Hardware	Features
Ground-truth	NovAtel SPAN	Yaw only
GNSS receiver	U-blox M8T	L1
GNSS Antenna	Patch antenna	GPS, GLONASS
odometer	CAN	velocity and yaw-rate

# Real world results

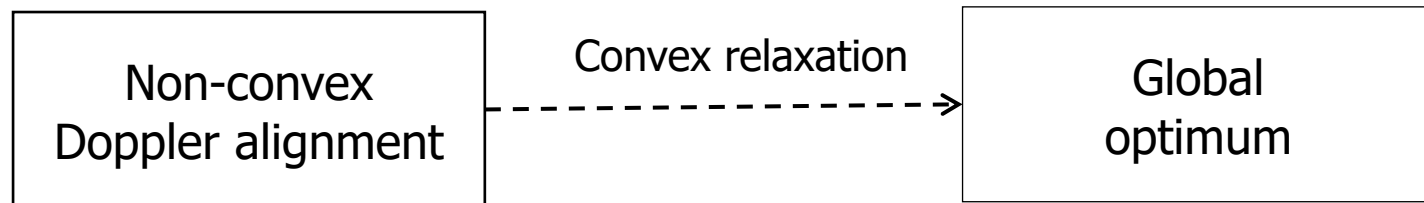
With 3 LOS satellites, motion in 60 s with two turning

<b>Yaw error [° ]</b>	<b>VOBA</b>	<b>GVINS</b>	<b>Ours</b>
Berlin	18.06	12.08	<b>12.07</b>
Berlin-10	17.12	200.32	<b>9.36</b>

1. With down-sampled sparse measurements, **GVINS** is stuck in **local optima** given random initialization, while VOBA and Ours is not affected;
2. **Our method** achieve the **best precision** compared to other methods.

# Take-aways

- Certifiably optimal GNSS/Local frame alignment
  - Certifiably correct alignment with at least **2 satellites** (under 3D motion + abundant constraints);
  - Both observability and **optimality** analysis is performed.
  - Codes and data will be **open-sourced**.
- Meaning of this work



# Future work

- Limitation

1. Ignore **measurement outlier** for simplicity;
2. The SDP **optimization accuracy** is limited;

- Future work

1. Develop **robust and certifiable using M-estimation and Lagrangian duality**;
2. Develop a fast and accurate certifier **without SDP optimization.**

# References

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# Open Source & Contact Information

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 Open-sourced software:  
LEO positioning; FGO-KFV comparison.



<https://github.com/Baoshan-Song/Certifiable-Doppler-alignment>

(mathematical derivation, codes, data)